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An Estimate of the Distribution and Stability Period of the Parameters of the Gamma Probability Model Applied to Monthly Rainfall Over Southeast Asia During the Summer Monsoon

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ABSTRACT—Following the Monte Carlo technique, we have obtained an estimate of the period of monthly rainfall that would provide approximate normality for the marginal distributions of the shape and scale parameters of the gamma model applied to monthly rainfall and an estimate of the stability period of these parameters. The long rainfall records (exceeding 130 yr) of Bombay, Calcutta, and Madras have been utilized. These three stations in India are from three distinctly different rainfall regimes during the summer monsoon. The marginal distributions of the shape and scale parameters appear to attain approximate normality when the period of monthly data approaches 75 yr. The stability period of the gamma model parameters (based on the specific criterion adopted) appears to be from 50 to 65 yr.

1. INTRODUCTION

In an earlier paper, Mooley (1973) showed that the gamma probability model, for which the probability density function is given by

$$P=\frac{x^{\gamma-1}e^{-x/\beta}}{\beta^{\gamma}\Gamma(\gamma)} \quad \text{for } x>0;\ \gamma,\ \beta>0$$

$$P=0 \qquad \text{for } x\leq 0,$$

is the most suitable among those Pearsonian models that show good fit to the Asian summer monsoon monthly rainfall. He used this model for the computation of monthly rainfall probabilities for the stations in the area. Fisher (1922) demonstrated that large-sample maximum likelihood (M.L.) estimates of the parameters of a distribution are normally distributed. He now must determine the sample size that would provide approximately normally distributed parameters of the gamma model applied to monthly rainfall. With this information, we could then determine whether reasonably accurate confidence limits on the estimates of the parameters can be obtained in the cases under consideration.

The probabilities of rainfall obtained on the basis of the gamma model are stable if the parameters of the model are stable. Thus, we also must determine the size of the rainfall sample necessary to obtain stable parameters.

A preliminary investigation on stability was made by Mooley and Crutcher (1968) for Bombay and Calcutta. They generated three random and one chronological rainfall sample each of size $n=10, 20, 30, \ldots, 140$ yr and computed $\hat{b}_n$ and $\hat{g}_n$, the M.L. estimates of the parameters of the gamma model applied to each sample. Stability was taken to be attained when, for each of the four samples, $\hat{b}_n$ came within the limits $\hat{b}\pm\sigma_{\hat{b}}$ $\hat{g}_n$ came within the limits $g\pm\sigma_{\hat{g}}$ and, thereafter, continued to lie within these respective limits for higher values of n. Here, $\hat{b}$ and $\hat{g}$ are the M.L. estimates, and $\sigma_{\hat{b}}$ and $\sigma_{\hat{g}}$ are their standard errors obtained for the total rainfall data for the station. The value of n at this stage was taken as the stability period. The preliminary estimates of the stability period for the summer monsoon months obtained from the study was between 70 and 90 yr. However, the criterion that $\hat{b}_n$ and $\hat{g}_n$ for all the four samples should lie within the adopted limits appears rather stringent. It may be mentioned that the sampling done by Mooley and Crutcher (1968) did not involve replacement.

Following a different approach and using the long record of rainfall of Bombay, Calcutta, and Madras located within three distinctly different rainfall regimes, we proposed to obtain broad estimates of the statistical distribution and the stability period of the M.L. estimates of the parameters of the gamma probability model applied to monthly rainfall during the summer monsoon.

2. METHOD OF APPROACH

Since the more general analytical approach is not possible, the Monte Carlo technique has been followed. This technique consists of statistical trials based on random sampling. The limitation of this technique, as mentioned by Schreider (1966), is that it gives approximate solutions of the problem. We feel that we can depend on the results obtained with the Monte Carlo method until it becomes feasible to evolve a more general method capable of giving more accurate solutions.

Monthly monsoon rainfall at Bombay, Calcutta, and Madras is available for 144, 132, and 148 yr, respectively. The samples of random numbers required in this experi-

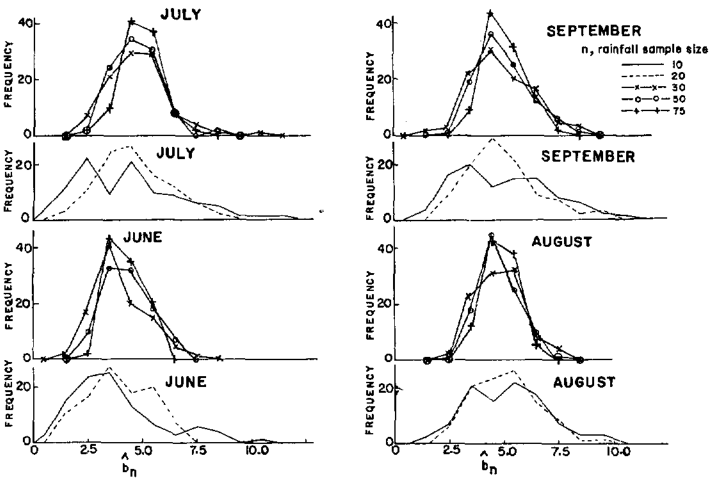

FIGURE 1.—Distribution of $\hat{b}_n$ for Bombay.

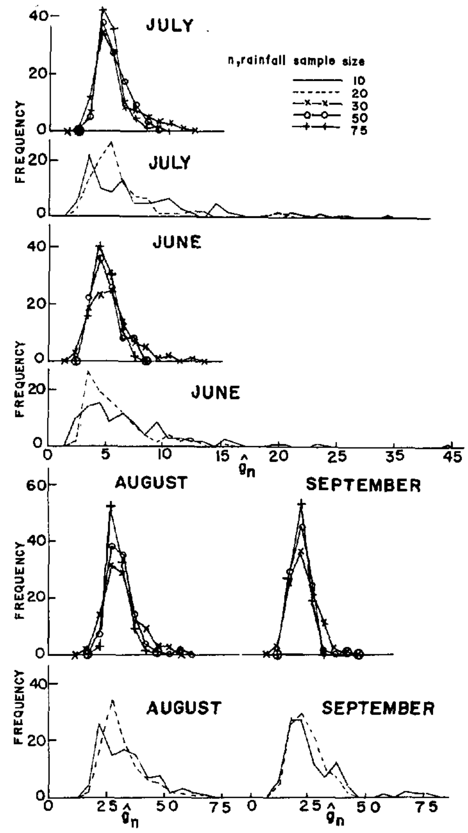

FIGURE 2.—Distribution of $\hat{g}_n$ for Bombay.

mental study were generated from the random digits provided by Owen (1962) and Fisher and Yates (1963). One hundred random samples, each of size 10, 20, 30, 50, and 75, were obtained for use with each of the three stations.

We then obtained a random rainfall sample corresponding to each sample of random numbers. Random sampling from each station's rainfall record was done with replacement since repetition of a rainfall value and occurrence of a value in close proximity to a value from the rainfall record are possible. Moreover, for nonrepetitive random sampling experiments, the available rainfall record is meager. Considering these points and the purpose of this investigation (i.e., to obtain a broad estimate of the statistical distribution and the period of stability of the M.L. estimates of the parameters), we felt that the procedure followed in getting the random rainfall samples is adequate. For each random rainfall sample for each of the three stations, $\hat{b}_n$ and $\hat{g}_n$, the M.L. estimates of the scale and shape parameters, respectively, of the gamma distribution were calculated. The subscript n denotes the size of the rainfall sample. This provides samples of size 100 for $\hat{b}_n$ and $\hat{g}_n$ corresponding to n equal to 10, 20, 30, 50, and 75 for each of the stations. Using these samples, we studied the statistical distribution and stability of the scale and shape parameters of the gamma model applied to monthly rainfall.

It may be mentioned that the estimate for γ has been obtained by solving the quadratic equation in γ as given by Thom (1958), and the estimate of β has been obtained by using the relation $\beta\gamma$=mean. These estimators have been termed Thom's estimators by Shenton and Bowman (1970*a*). In the Asian summer monsoon region, γ is greater than unity. For the three stations considered (i.e., Bombay, Calcutta, and Madras) and for the months of the summer monsoon season, γ vaired from 1.7 to 8.6. Shenton and Bowman (1970*a*) have shown that there is little difference in the variances of Thom's estimates and M.L. estimates for $\gamma \geq 1$. They have also shown that when γ exceeds unity, the skewness, $\sqrt{\beta_1}(=\mu_3/\mu_2^{3/2})$, and the kurtosis, $\beta_2(=\mu_4/\mu_2^2)$, for Thom's estimates differ little from those for the M.L. estimates of the parameters of the gamma distribution. Therefore, in the case under consideration, we used Thom's estimators as the M.L. estimators.

3. STATISTICAL DISTRIBUTION OF BETA AND GAMMA

Frequency polygons for the distributions of $\hat{b}_n$ and $\hat{g}_n$ for n=10, 20, 30, 50, and 75 have been plotted for each of the monsoon months for Bombay (figs. 1,2), Calcutta (figs. 3, 4), and Madras (figs. 5, 6). In general, these figures show that the marginal distributions of $\hat{b}_n$ and $\hat{g}_n$ for all the monsoon months are highly right-skewed for n=10 and 20 and substantially right-skewed for n=30. They are much less skewed for n=50 and 75. This suggests that the distribution may be normal or near normal for n=50 to 75. These distributions were tested for normality by testing the significance of g_1 or $\sqrt{b_1}$ $(=m_3/m_2^{3/2})$ and g_2 or $(b_2-3)[=(m_4/m_2^2)-3]$ and by applying the chi-square test of goodness of fit in the same way as was done by Mooley and Appa Rao (1971). The properties of the distributions of $\sqrt{b_1}$ and b_2 are important characteristics, and the percentage points spring from these. Pearson (1963) has attempted to approximate the distributions of $\sqrt{b_1}$ and b_2. He has shown that in normal sampling $\sqrt{b_1}$ (and to a less extent b_2) is reasonably well approximated by using the four moments (μ_1', σ^2, $\sqrt{\beta_1}$ and β_2), Johnson's S_u distribution, and t distribution (central for $\sqrt{b_1}$ and noncentral for b_2). The approximations are quite acceptable for nonextreme percentiles and more emphati-

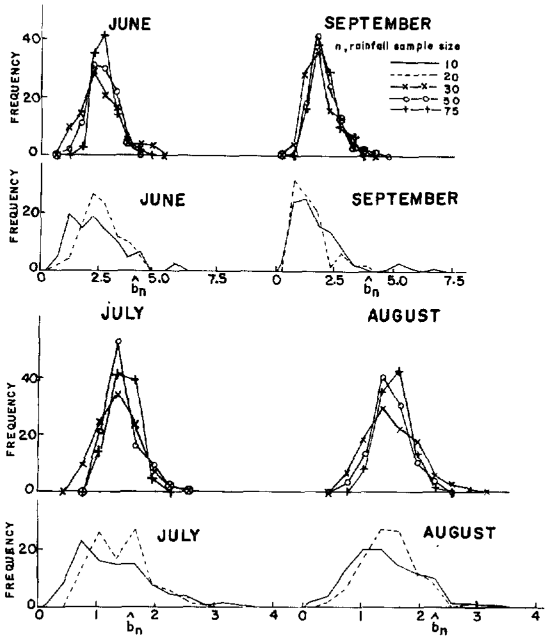

FIGURE 3.—Distribution of $\hat{b}_n$ for Calcutta.

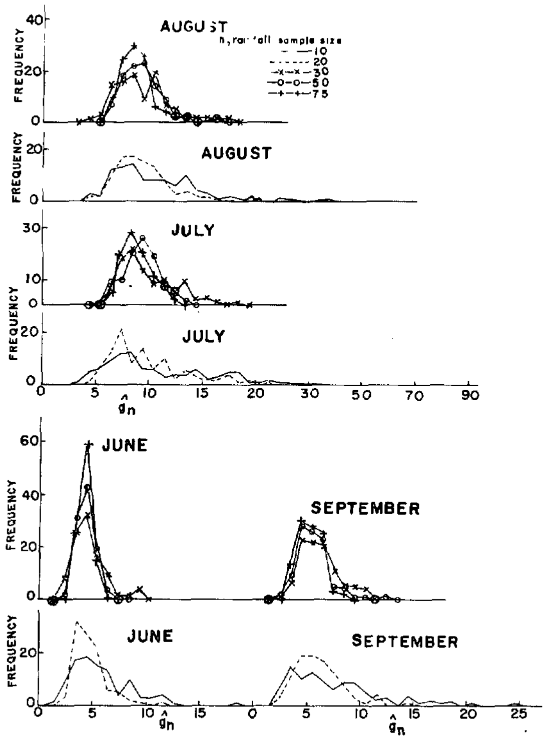

FIGURE 4.—Distribution of $\hat{g}_n$ for Calcutta.

cally for upper percentiles of $\sqrt{b_1}$ and b_2. The statistic b_2 is more difficult to deal with.

Since g_1 and g_2 can be positive or negative, a two-tailed test has been applied to test whether g_1 and g_2 are significantly different from zero. Five-percent and 1-percent levels of significance have been considered. The appropriate limits beyond which g_1 and g_2 become significant at these two levels have been obtained by using expressions for $E(g_1)$, var (g_1), $E(g_2)$, and var (g_2), given first by Fisher (1930) and later by Cramer (1946) where E

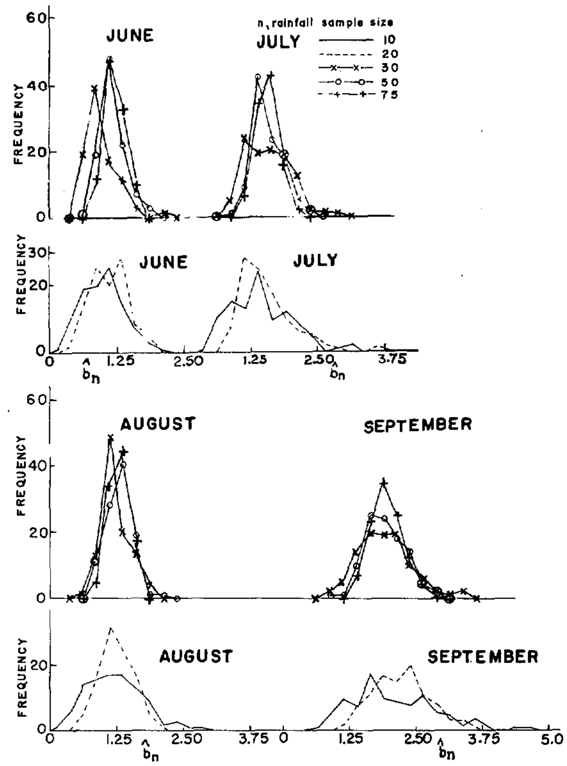

FIGURE 5.—Distribution of $\hat{b}_n$ for Madras.

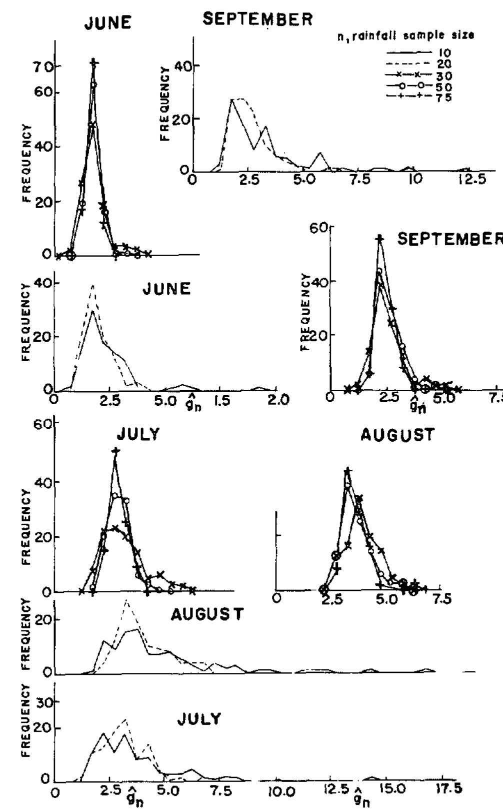

FIGURE 6.—Distribution of $\hat{g}_n$ for Madras.

denotes the expected value operator and var denotes variance.

The chi-square test of goodness-of-fit was applied after obtaining theoretical and empirical frequencies for eight

TABLE 1.—*Test for normality of the distributions of $\hat{b}_n$ and $\hat{g}_n$*

Station	Month	n, size of rain-fall sample	$\hat{b}_n$ g_1	$\hat{b}_n$ g_2	$\hat{b}_n$ Chi-square statistic (d.f.=5)‡	$\hat{g}_n$ g_1	$\hat{g}_n$ g_2	$\hat{g}_n$ Chi-square statistic (d.f.=5)‡
Bombay	June	50	0.35	−0.50	4.87	0.65†	−0.23	8.53
"	June	75	.30	−.89	14.59*	.44	−.21	5.18
"	July	50	.63†	.70	2.84	.58*	−.14	10.57
"	July	75	.37	.50	6.06	1.40†	3.78†	12.89*
						(0.51*)	(−0.18)	(11.64*)
"	Aug.	50	.30	.44	9.43	1.57†	5.20†	5.49
						(0.36)	(−0.78)	(4.85)
"	Aug.	75	−.10	−.55	6.61	1.02†	1.30†	14.27*
						(0.44)	(−0.95)	(12.02*)
"	Sept.	50	.64†	.14	8.89	1.30†	4.47†	7.44
						(−0.11)	(−1.37†)	(7.39)
"	Sept.	75	.52*	−.30	6.03	.22	−0.57	2.44
Calcutta	June	50	.16	−.25	3.15	.88†	1.30†	16.22†
						(.38)	(−0.53)	(15.58)†
"	June	75	.62*	−.26	6.64	.07	−.20	2.33
"	July	50	.79†	.25	12.15*	.11	−.33	2.30
"	July	75	.16	−.20	2.93	.66†	−.01	12.63*
"	Aug.	50	.30	−.08	1.75	1.10†	2.23†	4.65
						(−0.36)	(−0.36)	(4.54)
"	Aug.	75	.02	−.03	6.51	1.05*	1.65†	9.18
						(0.62*)	(0.08)	(8.24)
"	Sept.	50	1.05†	1.67†	12.71*	1.03†	2.30*	10.00
						(0.53*)	(−0.15)	(9.06)
"	Sept.	75	0.62*	−0.20	11.01	.46	.06	5.24
Madras	June	50	.74†	.36	12.02*	1.10†	3.45†	9.71
						(0.30)	(0.02)	(9.06)
"	June	75	.39	−.28	5.61	.66†	.54	4.92
"	July	50	.38	.05	14.31*	.79†	1.40†	6.64
						(.29)	(−0.44)	(5.40)
"	July	75	.14	−.57	2.00	.56*	−.01	9.90
"	Aug.	50	.28	.07	1.13	1.10†	1.37†	13.94*
"	Aug.	75	−.08	−.38	3.29	1.37†	2.78†	12.69*
						(0.58*)	(−0.61)	(11.95*)
"	Sept.	50	.30	.49	10.08	.97†	1.81†	9.71
						(.29)	(0.97*)	(9.07)
"	Sept.	75	.31	−.14	3.51	.75†	.78	7.86
						(.32)	(−.73)	(6.32)

*Significant at the 5-percent level
†Significant at the 1-percent level
‡d.f. is degrees of freedom

Note: The values of g_1, g_2, and the chi-square statistic recomputed after removing the noise-creating values of $\hat{g}_n$ are given in parentheses below the respective quantities.

intervals and computing the chi-square statistic,

$$\sum_{i=1} (f_i - O_i)^2/f_i,$$

where f_i and O_i are theoretical and empirical frequencies for the ith interval. Five-percent and 1-percent levels of significance were adopted for testing the significance of the chi-square statistic.

Since the significance of any one of the three statistics (i.e., g_1, g_2, and the chi-square statistic) does not imply significance of the other two, it is necessary to test the significance of all three and consider the distribution to be normal if none of these three statistics are significant.

Table 1 gives g_1 and g_2, Fisher's measures of skewness and kurtosis, respectively, and the chi-square statistic for the marginal distributions of $\hat{b}_n$ and $\hat{g}_n$ for $n=50$ and 75 for Bombay, Calcutta, and Madras for each of the summer monsoon months; the significance of each statistic is also given. Before considering this table, one must determine whether any of the isolated high values of $\hat{b}_n$ or $\hat{g}_n$ create any noise (Mooley 1973). A scrutiny of the values of $\hat{b}_n$

TABLE 2.—*Values of $\hat{g}_n \geq 3.5$ standard units*

Station	n, size of rainfall sample	Value of $\hat{g}_n$ in standard units for June	July	Aug.	Sept.
Bombay	50			4.95	4.92
"	75		4.47	3.88	
Calcutta	50	3.69		4.03	3.82
"	75			3.50	
Madras	50	4.31	3.69		4.09
"	75			4.20	3.52

TABLE 3.—*Cases with approximate normality of the distribution of $\hat{b}_n$ and $\hat{g}_n$*

Station	Month	n, size of rainfall sample
Bombay	June	75
"	July	75
"	Aug.	50 and 75
"	Sept.	75
Calcutta	June	75
"	Aug.	50 and 75
"	Sept.	75
Madras	June	75
"	July	50 and 75
"	Aug.	75
"	Sept.	50 and 75

and $\hat{g}_n$ shows that in no case did $\hat{b}_n$, when expressed as a standardized variable, equal or exceed 3.5 standard units, the limit adopted for rejection of the values. Details of the standardized variable, $\hat{g}_n \geq 3.5$ standard units, are given in table 2. These noise-creating values of $\hat{g}_n$ were deleted, and g_1, g_2, and the chi-square statistic were recomputed. The recomputed values one are given in parentheses below the appropriate statistics in table 1. The difference between the original and the recomputed g_1 and g_2 values is large, showing clearly that substantial noise was introduced by the isolated high values of $\hat{g}_n$. Hereafter, only noise-free values of g_1, g_2, and the chi-square statistic will be considered. In table 1, the single asterisk and dagger denote departures from normality significant at the 5-percent and 1-percent levels, respectively.

The test for normality in table 1 shows that approximate normality of the marginal distributions of both parameters is indicated for the cases listed in table 3.

It becomes possible to infer for each of the three stations that, in general for all the monsoon months, the marginal distributions of both the scale and shape parameters tend to approximately normal distribution as the rainfall sample size increases beyond 50 and approaches 75.

The preceding inference is expected to apply to the parts of Southeast Asia having monsoon rainfall regimes similar to those of Bombay, Calcutta, and Madras.

Bowman and Shenton (1968, 1970) and Shenton and

TABLE 4.—*Relative Mean Deviation (R.M.D.) of $\hat{b}_n$ and $\hat{g}_n$ from the median*

Station	Rainfall sample of size 10		20		30		50		75	
	$\hat{b}_n$	$\hat{g}_n$	$\hat{b}_n$	$\hat{g}_n$	$\hat{b}_n$	$\hat{g}_n$	$\hat{b}_n$	$\hat{g}_n$	$\hat{b}_n$	$\hat{g}_n$
Bombay										
June	0.485	0.678	0.309	0.363	0.234	0.255	0.181	0.197	0.159	0.154
July	.446	.608	.270	.340	.217	.251	.170	.182	.140	.143
Aug.	.304	.362	.232	.238	.181	.177	.158	.130	.127	.106
Sept.	.354	.365	.256	.228	.240	.175	.180	.141	.122	.115
Calcutta										
June	.403	.406	.245	.251	.242	.238	.156	.149	.136	.118
July	.423	.532	.270	.312	.199	.226	.150	.141	.125	.128
Aug.	.349	.369	.243	.246	.230	.223	.166	.150	.128	.125
Sept.	.559	.314	.350	.305	.291	.244	.230	.192	.205	.174
Madras										
June	.329	.387	.261	.247	.207	.198	.155	.132	.131	.109
July	.397	.436	.299	.274	.257	.227	.173	.141	.121	.115
Aug.	.366	.452	.218	.244	.163	.149	.138	.143	.118	.121
Sept.	.405	.468	.220	.241	.211	.203	.162	.149	.126	.116

Bowman (1969, 1970*b*, 1972, 1973), who have studied the properties of maximum likelihood estimators of the two-parameter gamma distribution and have derived expressions for the first four moments, also have considered the approach to normality of the marginal distributions of these estimators, $\hat{b}$ and $\hat{g}$. They found that, for $n \geq 70$, both maximum likelihood estimators of a strictly gamma distribution attain reasonable closeness to normality as judged by the significance of their skewness and kurtosis coefficients.

4. STABILITY PERIOD OF BETA AND GAMMA

The stability period depends on the criteria adopted. The criteria should, however, be fixed beforehand. Any statistic that changes value as n (the size of the rainfall sample) increases can be used to study the stability. In the present study, the statistic, relative mean deviation (R.M.D.) has been used. This is defined as the ratio of mean deviation from median/median. The criterion adopted for the stability period is that the R.M.D. attains a value of 0.15 and thereafter remains below this value. The stability period is the interpolated value of n at which the R.M.D. reaches the value of 0.15; then, for larger values of n, the R.M.D. remains smaller than 0.15.

The R.M.D. values were computed from the samples of $\hat{b}_n$ and $\hat{g}_n$ corresponding to n=10, 20, 30, 50, and 75 for each of the monsoon months and for each of the three stations. These values are given in table 4. The R.M.D. values decrease progressively as n increases. These values have been plotted on a graph (not shown), and a smooth curve has been drawn through the points to obtain the relationship between n and R.M.D. for each of the monsoon months and for each of the three stations. The values of n at which the R.M.D. attained the value of 0.15 have been noted from these graphs. In a few cases, it was necessary to slightly extrapolate the curve to obtain the value of n. Values of n corresponding to R.M.D.=0.20 were also interpolated to see how these compared with those corresponding to R.M.D.=0.15. Both of these sets of values of n are given in table 5. From the values of n corresponding to R.M.D.=0.15, the stability periods of the parameters were obtained for each of the stations and for each month by selecting the minimum period at which both the parameters attained stability. For example, for Bombay in June, the stability period for the scale parameter is 75 yr and that for the shape parameter is 80 yr; hence, 80 yr is considered to be the stability period of the parameters. The stability periods obtained in this way are rounded to the nearest five (table 6).

TABLE 5.—*Number of years after which the R.M.D. of $\hat{b}_n$ and $\hat{g}_n$ attain the specified values*

Station	Values of n when R.M.D. attained is 0.20		0.15	
	$\hat{b}_n$	$\hat{g}_n$	$\hat{b}_n$	$\hat{g}_n$
Bombay				
June	38	48	80	75
July	33	40	54	65
Aug.	25	25	43	38
Sept.	30	25	50	38
Calcutta				
June	38	39	51	50
July	30	34	50	44
Aug.	34	35	60	50
Sept.	60	47	85	80
Madras				
June	30	28	50	42
July	42	33	60	48
Aug.	23	28	35	48
Sept.	32	30	53	50

Table 6 indicates that, in general, the monthly rainfall data for a period of 50–65 yr are able to provide stable parameters of the gamma probability model when the R.M.D. of $\hat{b}_n$ and $\hat{g}_n$ attain the value 0.15. The stability periods for June rainfall of Bombay and September rainfall of Calcutta are larger. With the data given in table 4, stability periods corresponding to any other value of the R.M.D. can be obtained.

To decide whether the inference about the stability period can be applied to other stations in the field of the Asian summer monsoon, we must examine the relative variation of $\hat{b}$ and $\hat{g}$ over the area. The relative variation of $\hat{b}$ is $\sigma_{\hat{b}}/\hat{b}$, and that for $\hat{g}$ is $\sigma_{\hat{g}}/\hat{g}$. Using the expressions for the variance of $\hat{b}$ and $\hat{g}$ given by Thom (1958), we can express the relative variation as

$$\frac{\sigma_{\hat{b}}}{\hat{b}}\sqrt{\frac{\psi'(\hat{g})}{n[\hat{g}\psi'(\hat{g})-1]}}$$

and

$$\frac{\sigma_{\hat{g}}}{\hat{g}}\sqrt{\frac{1}{n\hat{g}[\hat{g}\psi'(\hat{g})-1]}}$$

where $\psi'(\hat{g})$=trigamma function$=\partial^2 \ln [\Gamma(\hat{g})]/\partial \hat{g}^2$.

TABLE 6.—*Stability periods (rounded to nearest 5 yr) of the parameters of the gamma distributions fitted to monthly rainfall*

Station	June	July	Aug.	Sept.
Bombay	80	65	45	50
Calcutta	50	50	60	85
Madras	50	60	50	55

TABLE 7.—*Analysis of the relative variation of* $\hat{b}$ *for stations in Southeast Asia*

	No. of stations having relative variation in the interval						
	>0.200 but ≤0.205	>0.205 but ≤0.210	>0.210 but ≤0.215	>0.215 but ≤0.220	>0.220 but ≤0.225	>0.225 but ≤0.235	>0.235 but ≤0.260
June	20	13	1	1	2	2	0
July	22	11	5	0	1	0	0
Aug.	18	14	5	2	0	0	0
Sept.	11	20	4	1	1	0	2

We computed these expressions for each station of the 39-station network used by Mooley (1973); however, for this purpose, we used only the last 50 yr of his data for fitting the gamma distribution.

The 50-yr period was practically identical over the area except for a few stations; therefore, we were able to compare the relative variations of $\hat{b}$ and $\hat{g}$ over the different parts of Southeast Asia.

Tables 7 and 8 give analyses of the relative variations of $\hat{b}$ and $\hat{g}$, respectively, over the area. These give the number of stations for which the relative variations lay within certain specified intervals. These tables indicate clearly that, for each of the summer monsoon months, about 90 percent of the stations in Southeast Asia have relative variations of $\hat{b}$ and $\hat{g}$ that lie within the narrow intervals of 0.200 to 0.215 and 0.185 to 0.200, respectively. The relative variations for $\hat{b}$ and $\hat{g}$ for Bombay, Calcutta, and Madras lie within the aforementioned intervals. Thus, the relative variation of the parameters $\hat{b}$ and $\hat{g}$ is fairly uniform over the whole area and differs little from that for the three stations. The measure adopted for obtaining the stability periods of the parameters for the three stations is a relative measure, defined by the ratio, mean deviation from the median/median. Therefore, it is reasonable to expect that the inference drawn about the stability period of the gamma parameters for the monsoon months for the three stations Bombay, Calcutta, and Madras can be extended to most stations in Southeast Asia.

5. CONCLUSIONS

1. The marginal distributions of the parameters of the gamma probability model applied to the monthly rainfall of Bombay, Calcutta, and Madras, India, during the summer monsoon season attain approximate normality as n, the size of the rainfall sample, approaches 75. This result could be applied to other stations in Southeast Asia having rainfall regimes similar to those at the three stations.

TABLE 8.—*Analysis of the relative variation of* $\hat{g}$ *for stations in Southeast Asia*

	No. of stations having relative variation in the interval					
	>0.170 but ≤0.175	>0.175 but ≤0.180	>0.180 but ≤0.185	>0.185 but ≤0.190	>0.190 but ≤0.195	>0.195 but ≤0.200
June	1	0	5	8	14	11
July	0	0	4	9	16	10
Aug.	0	1	3	13	15	7
Sept.	0	2	2	16	14	5

2. The parameters of the gamma probability model applied to the monthly rainfall of Bombay, Calcutta, and Madras obtained from a rainfall sample of size 50–65 are generally stable. This result is applicable to stations in the field of the Asian summer monsoon except possibly in cases of very dry regimes. It would not be advisable to use monthly rainfall samples of sizes smaller than 50 for obtaining stable parameters of the gamma model and, consequently, for obtaining stable rainfall probabilities.

3. If both approximate normality and stability are required together, then a rainfall sample size of about 75 would in most cases meet the requirement.

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